Appendix D

The Titius-Bode law

the very interesting
spacing of the
planets

In 1766, Johann Daniel Titius of Wittenberg in Germany formulated the modern version of the empirical rule now known as the *Titius-Bode law*. It was apparently done as a note that he added while translating a work from the French natural philosopher Charles Bonnet titled "Contemplation de la Nature" into German. The note, translated into English reads:

"Take notice of the distances of the planets from one another, and recognize that almost all are separated from one another in a proportion which matches their bodily magnitudes. Divide the distance from the Sun to Saturn into 100 parts; then Mercury is separated by four such parts from the Sun, Venus by 4+3=7 such parts, the Earth by 4+6=10, Mars by 4+12=16.

"But notice that from Mars to Jupiter there comes a deviation from this so exact progression. From Mars there follows a space of 4+24=28 such parts, but so far no planet was sighted there. But should the Lord Architect have left that space empty?

"Not at all. Let us therefore assume that this space without doubt belongs to the still undiscovered satellites of Mars, let us also add that perhaps Jupiter still has around itself some smaller ones which have not been sighted yet by any telescope.

"Next to this for us still unexplored space there rises Jupiter's sphere of influence at 4+48=52 parts; and that of Saturn at 4+96=100 parts. What a wonderful relation!"

In 1768, astronomer Johann Elert Bode published more or less the wording of Titius in the second edition of his introduction to astronomy: "Anleitung zur Kenntniss des gestirnten Himmels". He originally did not give credit to Titius, so the 'law' came to be known as Bode's law in the astronomy world. He did however, in later editions of his book, mention Titius as his source, hence the modern name of the law: the Titius-Bode law.

Bode did correctly note that the idea of Titius that the 'missing' planet may be a moon of Mars is incorrect and stated that a planet may yet be discovered at that distance. This eventually lead to the discovery of the largest asteroid, Ceres, at the predicted distance.*

*All the above information extracted from "BODE'S LAW AND THE DISCOVERY OF CERES" by Michael Hoskin, Churchill College, Cambridge.

Bode related the mean distances of the then known planets from the Sun to a simple mathematic progression of numbers. Start with the following sequence of numbers: 0, 3, 6, 12, 24, 48, 96, 192, 384, 768. Add 4 to each number, giving 4, 7, 10, 16, 28, 52, 100. Then divide each number by 10, giving 0.4, 0.7, 1.0, 1.6, 2.8, 5.2, 10.0. This gives the Bode distances from the Sun to successive planets in astronomical units (a.u.).*

*One astronomical unit (a.u.) = 149,597,870 km, the mean distance between Earth and the Sun.

The Titius-Bode law for distance D can also be formulated as follows:

$$D = 0.4 + (0.3 \times N) \text{ a.u.},$$

where $N = \text{integer}(2^{n-2})$ and n means the n^{th} planet from the Sun, counting in the asteroid Ceres. The series N works out to be 0, 1, 2, 4, 8, 16, 32, 64, 128, 256,.... The empirical law holds up well for planets up to Uranus, but breaks down for Neptune and Pluto, as is evident from table D.1.

Some Planetary Data in the Solar System					
n	**Planet**	D_m **(a.u.)**	D **(a.u.)**	P **(years)**	V_o **(km/s)**
1	Mercury	0.39	0.40	0.24	47.89
2	Venus	0.72	0.70	0.62	35.03
3	Earth	1.00	1.00	1.00	29.79
4	Mars	1.57	1.60	1.88	24.13
5	Ceres	2.77	2.80	4.61	17.90
6	Jupiter	5.20	5.20	11.86	13.06
7	Saturn	9.54	10.00	29.46	9.64
8	Uranus	19.19	19.60	84.01	6.81
9	Neptune	30.06	38.80	164.79	5.43
10	Pluto	39.53	77.20	248.54	4.74

Table D.1: Comparison between the correct mean distances of planets (D_m) and the predictions of Bode's law (D). $P = D_m^{1.5}$ is the period and $V_o = \frac{29.79}{\sqrt{D_m}}$ is the mean orbit velocity in a reference frame where the Sun is at rest.

Although Bode's law is merely empirical, there is today some serious considerations that it might have an underlying physical principle. No physical theory for the formation of the solar system could thus far derive such a principle in a rigorous way, but observations of planetary systems around other stars seems to suggest that they also follow something similar to Bode's law.

The following quote by John Gribbin from THE GUARDIAN, LONDON, illustrates the point: "The discovery of three planets orbiting a pulsar known as PSR B1257+12 has revealed a system with properties that almost exactly match those of the Inner Solar System, made up of Mercury, Venus and Earth. The similarities are so striking that it seems there may be a law of nature which ensures that planets always form in certain orbits and always have certain sizes; and it leads credence to the significance of a mathematical relationship that relates the orbits of the planets in our Solar System, which many astronomers have dismissed as mere numerology."

It is today accepted that Pluto is part of the Kuiper Belt, a huge disk-like plane of planetoids and comets that circle the Sun outside of Neptune's orbit. In 1951, astronomer Gerard Kuiper postulated the existence of such a 'belt', containing left-over debris from the formation of the solar system.

In 1992, a 150-mile wide body, called 1992QB1, was detected at the predicted distance of the Kuiper Belt. It was quickly followed by the detection of several similar-sized objects, confirming that the Kuiper Belt was real.

The Hubble space telescope detected many more smaller planetoids and also confirmed that many comets are circling the Sun in the Kuiper belt. This explained the existence of short-period comets (with periods of less than 200 years).*

*The Kuiper Belt is not to be confused with the Oort Cloud, which is postulated to be a spherical shell of comets at some 5000 a.u. It could be the source of the long-period comets.

Another interesting relationship between the periods of the planets is shown in figure D.1. The logarithms of the planetary periods against their sequential positions produce a nearly linear slope. The rotation period of the Sun near it's equator closely fits the position of 'planet zero'!

Although it may not look like it, for the planets up to Uranus, this method is far less accurate than Bode's law—the 'close fit' is an illusion caused by the logarithmic vertical scale. The actual deviation (in days) from the straight line 'law' is generally quite large, peaking at around $\pm 30\%$.

The relationship in figure D.1 also holds for the major moons of the larger planets, at least for those that have a reasonable number of major moons, like Jupiter and Uranus. Saturn has only one major moon (Titan) and likewise, Neptune has only one of any significance (Triton). Figures D.2 and D.3 show the plots for the periods of the major moons of Jupiter and Uranus respectively.

What does all this mean? Simulations of planetary formation are extraordinarily complex. It is thought that the solar system originated from a 'proto-solar nebula', a 'cocoon' of gas and dust that contracted under gravitational forces (perhaps aided by a nearby supernova explosion) to become denser at the center, forming the Sun. Such proto-solar nebulae are observed around other stars in our Galaxy.

The contraction caused the nebula to rotate and flatten out to form what is known as a 'proto-planetary disk'. Out of this disk the planets formed through accretion of gas and dust, passing through various phases.

It is thought that the process started with "dirty gas balls" that collided frequently to form 'planetesimals' (minute planets). Through a complex process of collisions, the planetesimals finally formed the planets as we know them.

This is an extreme oversimplification and as they say, 'the devil is in the detail'. Present day models produce neither the planetary spacings, nor the planetary sizes and compositions that we observe, so the 'right' model still needs to be found.

The computing power required to run more advanced models is exorbitant, so a representative model is probably still quite a distance into the future. The Internet contains many short articles about old and new theories of planetary formation, e.g., [space.com]. Just do a search on that website.

The log-linear plots are on the next page...

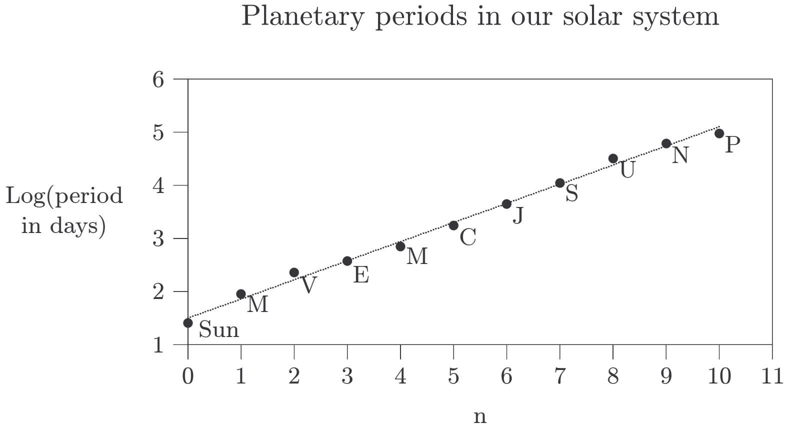

Figure D.1: A $\log_{10}$ plot of the planetary periods against the sequential positions of the planets. The Sun's surface is taken as 'planet 0', with the 'orbital period' equal to the rotation period at the Sun's equator. The straight line is the equation log(period)=1.5 + 0.35n.

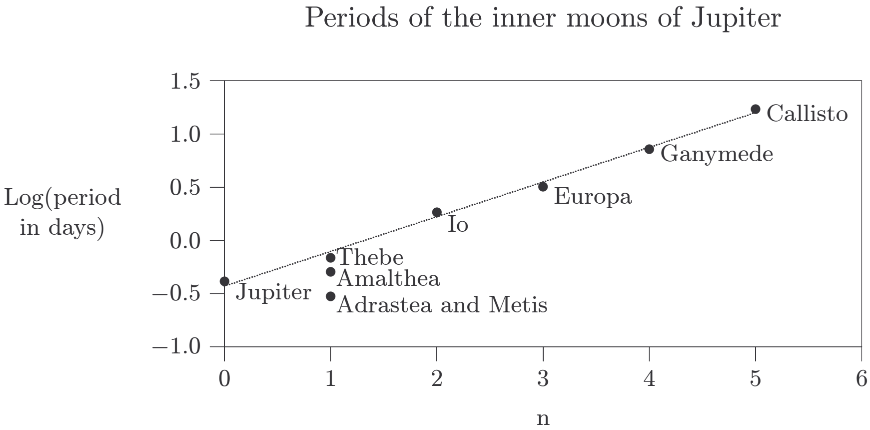

Figure D.2: A plot of the periods of the inner moons of Jupiter. Between Jupiter and Io, four asteroid-sized moons are found, but with irregular spacing. They do not fit the linear logarithmic 'law', while the four large 'Galilean' moons fit quite closely. Outside of Callisto, there are another eight small asteroid-like moons. They may possibly be passing objects that were captured by Jupiter.

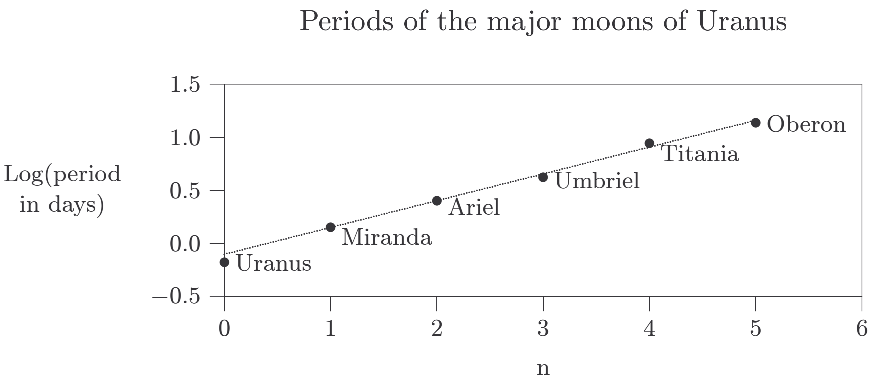

Figure D.3: The (log) periods of the major moons of Uranus. There are at least 10 very small moons between Uranus and Miranda (detected in 1986 by the spacecraft Voyager 2), but they seemingly do not require a separate interval.

Bibliography

[Coord.] Hamilton A. 1998. *Schwarzschild Coordinates*. Internet: http://casa.colorado.edu/

[Einstein] Einstein A. 1950. *Essays in Physics*. New York: Philosophical Library, Inc. Republished in 1996 as *The Theory of Relativity and other essays*. New York: CITIDAL Press.

[Faber] Faber, R.L. 1983. *Differential Geometry and Relativity Theory*. New York: Marcel Dekker, Inc.

[Ferguson] Ferguson K. 1999. *Measuring the Universe*. London: Headline Book Publishing.

[GP-B] NASA 2004. *A Pocket of Near-Perfection: Gravity Probe B*. Internet: http://science.nasa.gov/headlines/y2004/26apr-gpbtech.htm

[Gribbin] Gribbin J. and Rees M. 1991. *Cosmic Coincidences*. London: Black Swan.

[Guth] Guth A. and Steinhardt P. 1990. *The Inflationary Universe*. Scientific American Jan. 1990.

[Ibison] Ibison M., Puthoff H. and Little S. *The speed of gravity revisited*. Austin: Institute for Advanced Studies (also available on Internet).

[Mitton] Mitton, J. 1991. *A Concise Dictionary of Astronomy*. Oxford University Press.

[MTW] Misner C.W., Thorne K.S. and Wheeler J.A. 1973. *Gravitation*. San Francisco: W.H. Freeman and Company.

[Pathria] Pathria R.K. 2003. *The Theory of Relativity, 2nd ed.*. New York: Dover Publications Inc.

[Peacock] Peacock J.A. 1998. *Cosmological Physics*. Edinburgh University Press.

[Peebles] Peebles P.J.E. 1993. *Principles of Physical Cosmology*. Princeton University Press.