Chapter 13

The Einstein-de Sitter Universe

The
simplest
model

Way back in 1932, Einstein and de Sitter presented the 'standard' model of the cosmos to the world.* It is the simplest possible model and has been

*This must not be confused with the 'De Sitter model', which was an earlier effort of De Sitter alone [Peebles, chapter 5].

the favorite amongst cosmologists until the 1980s. In a way it was the 'de facto industry standard' for over 50 years.

In short, this model started in an extremely dense state, much like the elementary 'Escher model' discussed in the introduction. The expansion rate must have been extreme in the beginning, but fine tuned so that the gravitational pull of the matter in the universe was precisely balanced by the kinetic energy of expansion. In other words, potential energy and kinetic energy of expansion had to balance out, with a nett energy of zero.

This fine balance had to be maintained until the present and, according to this model, will be maintained forever. This chapter will examine the major properties of the 'Einstein-de Sitter model'. More modern variations of this model will be examined in later chapters.

13.1 Einstein-de Sitter spacetime

What must be stressed is that particles, atoms, molecules and later congregations of matter are not expanding into pre-existing space. It is space itself that is expanding.

When we draw a diagram of Einstein-de Sitter spacetime, as shown in figure 13.1, it appears as if the expansion were driving matter outwards at speeds exceeding that of light. But light itself was being driven outwards with the expansion, so that relative to the (stretching) fabric of spacetime, light was still moving at its normal speed and matter were always moving at less than the speed of light.

The curve in figure 13.1 shows the "edge" of the observable universe as it would have looked in one space and one time dimension over the history of the universe. Actually, 'looked' is not a good word choice, since no observer could have 'seen' the 'edge' of the universe like that.

Since we are limited to observe the universe by means of light and other electromagnetic radiation, which do not have infinite propagations speed, we see a completely different picture. We can however use observational data to draw such a graph, but it will always be somewhat model dependant—in this case the Einstein-de Sitter model.

The scales of time and space shown is further dependant upon the value of the Hubble constant, H_0, for which 50 km/s/Mpc was used here, simply because it was the favorite value for most of the time of Einstein and de Sitter.

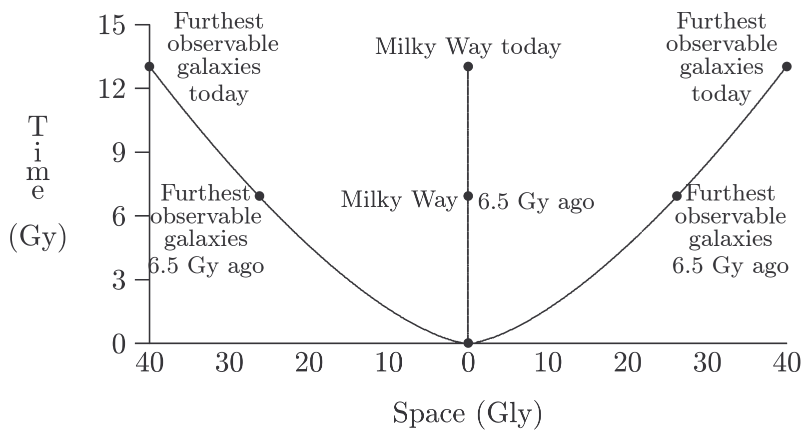

Figure 13.1: A spacetime diagram for the observable universe according to the Einstein-de Sitter model, where the expansion curve is parabolic. The shown positions of the remote galaxies are not where they are observed, but their actual (presently unobservable) positions.

We will now briefly look at what 'observable' space in an expanding universe means.

13.2 Observable space

Our main observational method of the universe is through electromagnetic radiation (photons) of various wavelengths. Unlike material objects, which can be stationary in space, photons cannot be stationary. They always have to move at the speed of light in some or other direction in space.

If we trace the path of a photon that was tranmitted in our direction from near the edge of the (expanding) observable universe at a very early time, it will follow one side of the teardrop-shaped curve shown in figure 13.2. The reason for the shape is that when the expansion rate was very high, the photon would effectively have been dragged away from us (the central world line).

Since the photon always moves at precisely the speed of light through local space, it will move away from the 'edge' and eventually find itself in a region where the rate of expansion is slow enough for it to start approaching the central world line. Eventually it will reach us and can be detected.

Photons from the very edge of the observable universe will take the full age of the universe to reach us. Areas further than the current edge will reach us some time in the future. From areas inside the current edge we will observe a continuous stream of photons, which will be elaborated on in the next paragraph.

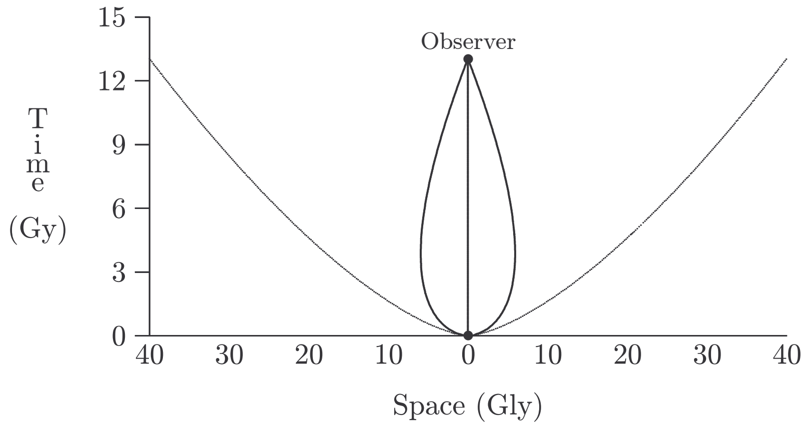

Figure 13.2: The teardrop-shaped curve in the centre represents the paths of two photons, transmitted in opposite directions from the edge of the observable universe when the universe was very, very young. They are presently being observed in the Milky way for the first time. For every instant in time, there is a slightly different 'teardrop', representing the path of another pair of photons.

Essentially, their are an infinite number of 'teardrops', one for every instant of observation. However, at one instant, we can in principle observe an infinite number of photons, coming from different distances, all following the same 'teardrop' path.

In just one space dimension, observing multiple photons may be difficult, if not impossible. In more than one dimension, the problem largely dissappear. We can then resolve different photons arriving simultaneously from slightly different directions.

In figure 13.3, the parabolic expansion curves for the spacetime of two galaxies at intermediate distances are drawn, one at one third and one at two thirds of the distance to the end of observable space.

We observe them as they were when the universe was 0.46 Gy and 3.8 Gy old respectively, as shown in the figure. Light took $\approx 13 - 0.46 \approx 12.5$ Gy and $\approx 13 - 3.8 \approx 9$ Gy respectively to reach us from those galaxies. This is also their respective distances in *light travel time*.

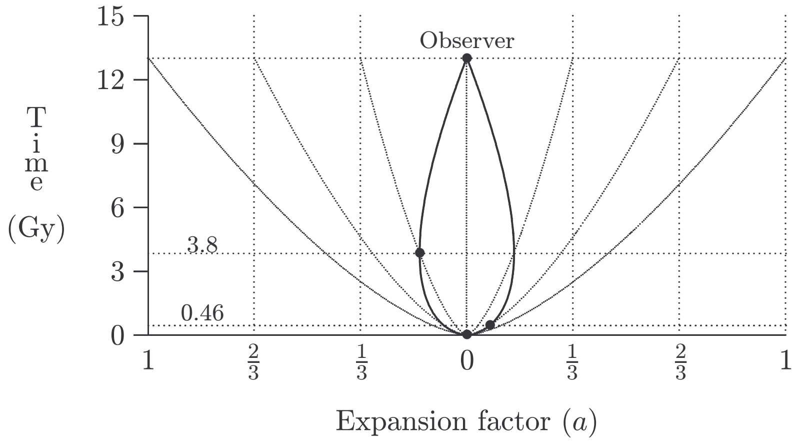

Figure 13.3: The spacetime expansion curves of two pairs of galaxies, presently at 10 and 20 Gly from us respectively. Where the curves intersect the 'teardrop' is where the galaxies were when we observe them today—in principle at least—they may be too far to be observed in practice.

13.3 Standard expansion model concepts

In order to comprehend cosmological models, we must first firmly establish some basic concepts around the Hubble constant and it's units. The Hubble constant H_0 (pronounced 'H naught' or 'H zero') is defined as the apparent speed of recession of a distance object per unit distance.

In the SI convention, the units of H_0 should really be (metres/second)/meter, giving second^{-1}. This would give an extremely small value for H_0, so Edwin Hubble decided to use the units km/s/Mpc, giving a 'friendly' range of values, between 50 and 100 km/s/Mpc.*

Cosmologists further define a dimensionless *Hubble parameter* h. It has

*One Mega-parsec (Mpc) is about 3.3 million lightyears.

the value $h \equiv H_0/(100 \text{ km/s/Mpc})$, with an original 'best fit value' around 0.5, meaning $H_0 \approx 50$ km/s/Mpc.

The parameter h is often used in conjunction with other dimensionless cosmological parameters, to make them valid for any value of the Hubble constant H_0, especially when such parameters are extracted from observational data. More about that later.

It must be noted that the Hubble constant is not necessarily constant, because it must have been much higher in the past. So H_0 is referred to as the present Hubble constant or also the local Hubble constant. At other times, the value is denoted by just H or by $H(t)$, meaning the 'time varying Hubble constant'.

In the Einstein-de Sitter expansion model, the mutual gravity of all the matter in the universe must be balanced by the expansion rate of the entire universe. this must be done in such a way that the expansion rate is just high enough to prevent an eventual re-collapse of the universe.

This requirement demands a very specific expansion law. In imitation of the escape velocity: $dr/dt = \sqrt{2GM/r}$, the rate of change of the expansion factor equals

$$\frac{da}{dt} = \sqrt{\frac{H_0^2}{a}}, \tag{13.1}$$

if appropriate units are chosen. H_0 is the Hubble constant and a the dimensionless (time varying) expansion parameter* at time t. Here H_0^2 is

*Recall that a is defined to be unity at the present time and 0.5 when the observable universe was half it's present size.

equivalent to $2GM$, a constant energy, because H_0 represents the 'velocity' of expansion and energy is proportional to velocity squared.

This is the *expansion law for the Einstein-de Sitter universe*. It can also be written as

$$dt = \frac{a^{\frac{1}{2}}}{H_0} da$$

and integrated against a, to find

$$t = \frac{1}{H_0} \int a^{\frac{1}{2}} da = \tfrac{2}{3} \frac{a^{\frac{3}{2}}}{H_0} + \text{ constant.}$$

It is assumed that $t = 0$ for $a = 0$, making the constant of integration zero. In standard textbooks, e.g., [Peebles, Peacock], this relationship is usually written as

$$H_0 t = \tfrac{2}{3} a^{\frac{3}{2}}, \tag{13.2}$$

which was used to plot the expansion curves of the preceeding figures of this chapter.

Since a is a dimensionless parameter, the above implies that H_0 has the units (time)$^{-1}$. If time is measured in Gy, then H_0 has the units Gy^{-1}.

The conversion from the standard units for H_0 (km/s/Mpc) to Gy^{-1} is a factor $\frac{1}{978}$ in appropriate units. Cosmologists seem to happily live with this 'dual use' of the symbol H_0, but engineers normally don't.

The author prefers to show the difference more clearly by defining a normalized Hubble constant

$$\bar{H}_0 \equiv \frac{H_0}{978}\ \mathrm{Gy}^{-1},$$

as will be used in equations in the rest of this book. Because at present, $a = a_0 = 1$, equation 13.2 immediately gives us the present age of the universe, as predicted by the Einstein-de Sitter model:

$$t_0 = \tfrac{2}{3}\frac{1}{\bar{H}_0}\ \mathrm{Gy}. \tag{13.3}$$

In the early days, when H_0 was taken as around 50 km/s/Mpc (or $\bar{H}_0 = 0.051$ Gy^{-1}), this gave an age for the 'standard' universe of about 13 Gy. We will see in later chapters that a new 'standard' model has been developed, giving about the same age, but with a Hubble constant around 0.07 Gy^{-1}.

In the Einstein-de Sitter model, this Hubble value gives an age below 10 Gy, which is incompatible with other observational data. From equation 13.2, we also have the following useful relationship

$$\frac{a}{a_0} = a = \left(\frac{t}{t_0}\right)^{\frac{2}{3}}, \tag{13.4}$$

telling us that at as we look back in time, say to when the universe was an eighth of it's present age, the expansion factor was a quarter of what it is today, because

$$a = \left(\frac{1}{8}\right)^{\frac{2}{3}} = \frac{1}{4}.$$

An expansion factor $a = 1/4$ means that the present visible universe was then one quarter of it's present size.

13.4 Redshift

Engineers are mostly familiar with the Doppler shift caused by objects moving through the air or through space (like radio waves). Redshift is essentially the same thing, but in cosmology it may have two origins.

The first is like Doppler shift, where a source is moving through space relative to us. It becomes a blueshift if the source is moving towards us. Then there is cosmological redshift caused by the expansion of space.

Since photons move through space as electromagnetic waves, it is reasonable to accept that their wavelengths stretch with space. If a photon was emitted into space when the expansion factor was $a = 0.5$, then today, with expansion factor $a = 1$, all distances have stretched by a factor $\frac{1}{0.5} = 2$ and so has the wavelength of the photon.

If the photon had a wavelength λ when emitted, it will now have a wavelength $\frac{\lambda}{a} = 2\lambda$. The increase in wavelength will be $\Delta\lambda = \frac{\lambda}{a} - \lambda = \lambda$ in this case, with $a = 0.5$. Expressed as a fraction of the original wavelength:

$$z = \frac{\Delta\lambda}{\lambda} = \frac{1}{a} - 1 \tag{13.5}$$

which will be unity in this case. The parameter z is called the cosmological redshift We can also express the expansion factor a as a function of redshift z:

$$a = \frac{1}{z+1} \ . \tag{13.6}$$

Equations 13.5 and 13.6 are very important relations in cosmology. Since the present value of a is unity, light emitted very recently will have a redshift approaching zero.

If the expansion factor was very small when the photon was emitted, the photon's redshift would be very large, as is clear from figure 13.4. Here the redshifts for the galaxies that we worked with before (fig. 13.3) are shown against the expansion factor.

It is interesting to note that turn-of-the-millennium technology only allowed observation of galaxies up to just over $z = 6$. The only observations at significantly larger redshifts were the cosmic microwave background (CMB) radiation, weighing in at just over $z = 1000$. More about the CMB later in this chapter.

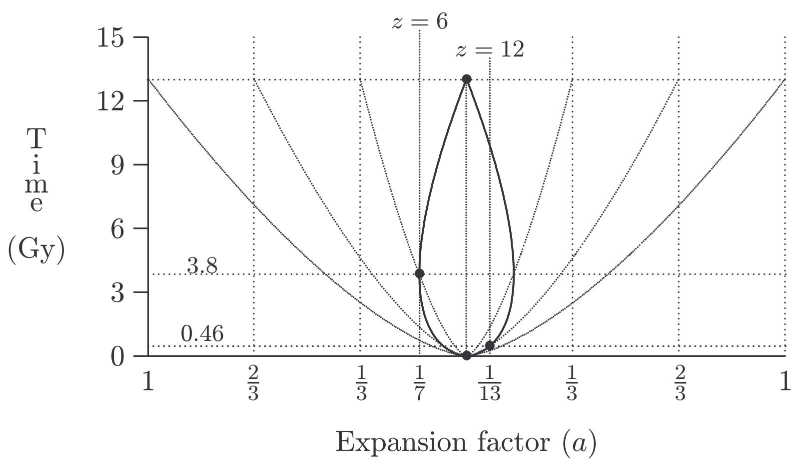

Figure 13.4: The expansion curves against expansion factor, allowing the redshift of galaxies to be calculated where the teardrop and the curves intersect.

13.5 The other 'Hubble quantities

Hubble time is the inverse of the Hubble constant, in appropriate units of course. Cosmologists sometimes confuse people outside their trade by stating the Hubble time as

$$t_H = \frac{1}{H_0} = \frac{9.78}{h} \text{ Gy},$$

implying, but not saying, that H_0 is expressed in Gy^{-1} and $h = 100$ km/s/Mpc. One should rather avoid this potential confusion and use the normalized Hubble constant, simply stating

$$t_H = \frac{1}{\bar{H}_0} = \frac{978}{H_0} \text{ Gy}.$$

It is essentially the time it would have taken the present observable universe to expand from near zero size to it's present size, given that the expansion rate was always the same as today.

If we take the 'old' value of $\bar{H}_0 \sim 0.05$, it means that $t_H \sim 20$ Gy, which is a 'characteristic' timescale, but not the age of the universe. We have seen above that in the Einstein-de Sitter model, the age of the universe is two-thirds of the Hubble time.

There are more modern models that utilize the same Hubble time, but yields a different fraction than two-thirds, caused by a different expansion law. These will be dealt with later in this chapter.

Hubble distance is simply the distance that light can travel in the Hubble time. Because the speed of light is 1 lightyear per year, the value of the Hubble distance is the same as Hubble time if expressed in the units of Gly.

The Hubble distance, also called the Hubble radius (r_H), is a characteristic scale for the universe. Like in the case of the age of the universe, the radius of the observable universe is not equal to the Hubble radius.

For the Einstein-de Sitter model, it is again two-thirds of the Hubble radius, based on light travel time. Based on co-moving coordinates, the radius of the observable universe is about 40 Gly, assuming $\bar{H}_0 \sim 0.05$ Gy^{-1} (or 50 km/s per Mpc) for now.

13.6 Look-back time

When we observe a distant celestial object, the radiation that reaches us traveled along the surface of the "cosmic teardrop". The space distance that the radiation traveled is however not the distance as measured along the surface of the teardrop, because that distance includes a component of time.

The space distance that the light traveled is called the look-back distance, which is the same as the look-back time, in appropriate units, of course.

The look-back time is $t_0 - t$, where t_0 is the present time and t the time when the radiation left the object.

For the standard Einstein-de Sitter universe, we have seen that $a = (t/t_o)^{\frac{2}{3}}$, or $t = t_0 a^{\frac{3}{2}}$, so that

$$t_0 - t = t_0(1 - a^{1.5}).$$

Since $a = \frac{1}{1+z}$ (eq. 13.6) and $t_0 = \frac{2}{3\bar{H}_0}$ (eq. 13.3), we can express the look-back time $t_0 - t$ in terms of the redshift z as

$$t_0 - t = \frac{2}{3\bar{H}_0}\left(1 - (\frac{1}{1+z})^{1.5}\right) \ , \qquad (13.7)$$

perhaps one of the most used equations in 'standard' cosmology—at least in the pre-1980 era. It gives the (light travel) distance to a remote cosmic object in terms of two measurable quantities, the redshift and the Hubble constant.

Let us use equation 13.7 to calculate the look-back time to a galaxy at redshift $z = 6$, taking $\bar{H}_0 = .05$ Gly^{-1}.

$$t_0 - t = \frac{2}{3 \times 0.05}\left(1 - (\frac{1}{7})^{3/2}\right) \approx 12.6 \text{ Gly}.$$

13.7 Cosmic microwave background radiation

If the universe started out in an almost infinitely dense state (the 'big bang'), the temperature must have been extreme. As determined by modern theory, the temperature must have been in the order of 10^{15} degrees Kelvin, e.g., [Smoot], by the time the normal particles that makes up matter emerged.

However, during the first 400,000 years or so, radiation was so strongly coupled to the elementary particles, that the universe was not transparent. In effect, the photons were scattered by the charged elementary particles. The universe was opaque.

When the temperature dropped enough so that electrons could bind with nuclei, neutral atoms were formed, allowing radiation to become free to move through space. The universe became transparent. It is called the time of 'last scattering', with a temperature of around 3,000 degrees Kelvin.

Today, astronomers observe this radiation as the cosmic microwave background (CMB), at about 2.7 degrees Kelvin (equivalent black body temperature). For a detailed account of how the CMB was accurately charted, refer to [Smoot].

So how much has the universe expanded since last scattering? The relationship between temperature and the expansion factor happens to be linear, so that the expansion factor must have increased by about a factor $3,000/2.7 \sim 1,100$.

This gives an expansion factor at last scattering of $a_{ls} \sim 9.1 \times 10^{-4}$ and a redshift of $z_{ls} = (1 - a)/a \sim 1,100$. Astronomers and cosmologists usually round this to $a_{ls} \sim 10^{-3}$ and $z_{ls} \sim 1,000$.

An interesting question: why can we continously observe the CMB, or in other words, why have those photons from the last scattering not whisked past us, never to be seen again? The answer is that there are apparently much more universe than what we can observe today.

The big-bang happened everywhere simultaneously. As time goes on, we observe further and further regions of space as they were at last scattering. If the universe is infinite in size, the CMB will never become unobservable, but it will be observed at lower and lower temperatures (larger redshift) as time goes on.

If the universe happens to be finite but not closed on itself (i.e. it is larger than today's observable universe, but it has boundaries or an edge), then there may come a time when there will be no CMB as we know it today—when the last CMB photons have whisked past us, never to be seen by us again.

13.8 Summary

We now have a feeling for the Einstein-de Sitter model in terms of the expansion law and Hubble's constant. We have seen how the redshift relates to the expansion factor, the age of the universe, look-back time and the CMB.

All was done in terms of the standard 'flat' model. It is now time to look at various other expansion models. As we have seen previously, the simple flat model takes as an assumption that the kinetic energy of expansion exactly balances out the potential energy caused by the mutual gravitational pull of the matter of the universe. What if they do not balance out?