Chapter 15

Inflation

an engineer's view
of
inflationary expansion

If the Friedman model is taken to it's lower limit (when $a \to 0$), the expansion rate becomes extremely large. From what we have discussed in the previous chapter and specifically equation 14.9, it is clear that under this condition the radiation density (Ω_r) dominates the expansion equation, i.e.

$$\dot{a}_{(r\to 0)} \to aH_0\sqrt{\Omega_r/r^4} \to \infty.$$

This very fact presents a puzzling problem to cosmologists; not so much the very rapid expansion rate, but rather the so called 'horizon problem' that it creates. In short, the horizon problem means that if space expanded at a tremendous rate right from time zero, light and other radiation that moves *through* space could not have kept up with the expansion—only parts of space that were in very close proximity to each other could have exchanged any sort of radiation information.

However, if we look at space today we find that the cosmic microwave background radiation has very, very close to the same temperature everywhere. The question is, how did parts of space that could not (ever) have exchanged radiation, ended up with this uniform temperature?

15.1 Guth's insight

In 1979 Alan Guth published a ground breaking paper that showed how the very, very early universe could have started with a very slow expansion rate and then went through a period of exponentially increasing expansion.* After some time the exponentially increasing expansion rate went over to

*A good summary is found in [Guth].

the conventional Friedman expansion, where the rate of expansion started to slow down. In short, he introduced a time variable cosmological 'constant' that acted like an anti-gravity force that could have 'inflated' the universe at a tremendous rate.

A phase transition then stopped the inflationary epoch and the normal 'flat' expansion dynamics took over. Later analysis showed up flaws in the argument,* but the main elements survived in essence. A physical

*It had to do with the 'first order' phase transition that Guth proposed and was replaced by Linde (1982,1983) and also by Steinhardt (1982) with a 'second order' phase transition.

treatment of the subject of inflation is outside the scope of this book, firstly because it is technically very intimidating and secondly because the experts do not seem to agree on the model.

In the standard Friedman model there is general agreement on the model and just disagreement about some of the parameters in it. For the inflationary epoch, however, there is no agreement on the model's characteristics, let alone it's parameters. In this chapter, we will attempt to add some value for the reader by using an engineering approach—take the simplest model of the physicists at face value and look at it's implications.

15.2 The simplest model

In it's simplest form, inflation theory postulates that the time varying Hubble 'constant' $H(t)$ might actually *have been constant* [Peebles, page 407], in the very early universe, i.e.,

$$H(t) = \dot{a}/a = \text{constant}, \tag{15.1}$$

meaning that the expansion rate $\dot{a}$ was directly proportional to the expansion factor a. Now this may look innocent enough, but a simple 'everyday' example shows that it is not so ordinary.

Make a spacecraft accelerate from rest in such a way that it's velocity $\dot{s}$ is always directly proportional to s, the distance traveled, i.e., $\dot{s} = ks$, where k is some positive constant. For simplicity, let k be unity, so that at one meter distance, the speed must be 1 m/s, after 10 meters, 10 m/s and so on.

From this we can suspect that we are dealing with an exponential situation. The faster the craft travels, the less time it takes to traverse 1 meter distance and the less time it has to make that next 1 m/s speed change. This relationship between distance and time is indeed exponential and can be expressed as

$$s = be^{\alpha t}, \tag{15.2}$$

where for this example, $\alpha = 1$ and b is an arbitrary constant (a scale factor), which cannot be determined from the information we have.* The

*Many different initial accelerations can satisfy the conditions set.

curve is shown in figure 15.1, where $b = 10^{-6}$ was chosen, so that it gives a convenient scale.

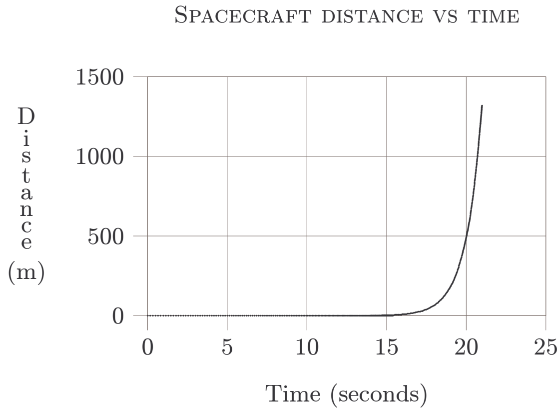

Figure 15.1: A plot of the distance that the spacecraft traveled against time if it's speed has to be directly proportional to the distance traveled. Note how close the distance (and by definition, also the speed) remains to zero, up to about 15 seconds and then quickly increases until it will approach infinity as times goes on.

The distance remains small for the first 15 seconds or so and then diverges relatively quickly to a large value, increasing ever faster. The 'time constant for divergence' (which we shall call t_d) is obtained by taking the natural log of the right hand side of the equation and setting it to zero, i.e. $\ln(be^{\alpha t_d}) = \ln(b) + \alpha t_d = 0$. Therefore

$$t_d = \frac{-\ln(b)}{\alpha} = \frac{-\ln(10^{-6})}{1} = 13.82 \text{ sec.}$$

In a way the spacecraft's speed control starts to go unstable at 13.82 seconds into the mission.

Obviously, a real spacecraft can never behave exactly like this, no matter how badly us engineers design the stability of the control loop. We just do not have the propulsion system to cope with this instability! But according to inflation theorists, the very early universe could have had the required energy of 'propulsion'—at least for a *really* short time.

Assuming that equation 15.1 above is valid, cosmological inflation can be viewed as analogous to the 'unstable' spacecraft—after some time of slow expansion there will be an exponential increase in the expansion rate. Let r be the spatial radius of a spherical piece of space that represents our observable universe at a time near $t = 0$. By analogy to equation 15.2 above, the radius r will change with time t as

$$r = be^{\alpha t}, \tag{15.3}$$

The inflationary epoch

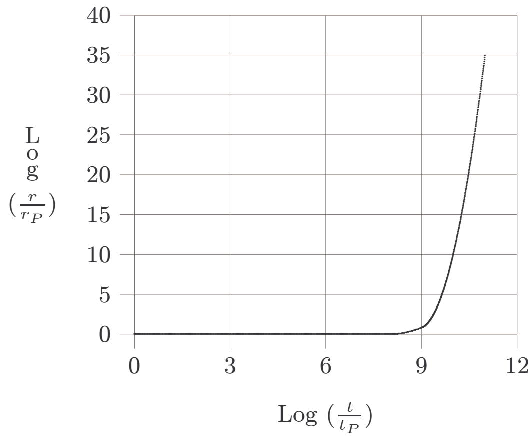

Figure 15.2: A log-log plot of r against time for the inflationary epoch, where the expansion rate ($\dot{r}$) is proportional to the radius (r). Like in the case of the 'unstable' spacecraft, the expansion goes unstable rather rapidly. r_P is the Planck length and t_P the Planck time.

where b is some constant (a scale factor) and $\alpha = \sqrt{\frac{8}{3}\pi G \rho_v}$, the energy density of the vacuum, which is assumed to be constant.

Figure 15.2 shows a plot for the first 12 time units or so. The Planck time ($t_P \approx 10^{-43}$ seconds) is used as unit time and the Planck length ($r_P \approx 10^{-43}$ lightseconds) as unit length.

We assume that at time t_P the radius r was equal to r_P, the smallest size that has any physical meaning. It looks much like the plot for the spacecraft, even though this plot is done on a log-log scale. See box on page 200 for more on the units and constants used.

We find that in the first 9 time units or so, the expansion rate was very slow. Then at about 9 time units the expansion rate 'exploded'. At about 11 time units, according to the inflation theorists, the extreme input of vacuum energy stopped because the false vacuum went through a phase transition back to the classical vacuum.

The expansion then takes on the classical shape, where the expansion rate slows down under the mutual gravitational attraction of the matter and radiation.

Since the expansion rate was very slow near time zero, it allowed all parts of the small sphere to be 'causally connected'. This means that any influence, propagating at the speed of light, could reach from one end of the sphere to the other end—in fact it could do so many times over. This made the sphere very homogeneous and having the same temperature everywhere (almost*

*Quantum fluctuations is thought to have caused small deviations from the mean temperature.).

Then, in the incredibly small time interval of about 10^{-32} seconds, the radius r grew from less than 10^{-28} metres to in the order of 1 metre. The expansion rate $\dot{r}$ reached something in the order of 10^{24} times the speed of light.*

*The expansion rate of space is not constrained by the speed of light—only movement through space is constrained.

The small quantum fluctuations in the mean temperature and density that existed at 9 time units were magnified immensely and due to the tremendous rate of expansion, no further causal connection between regions were possible. Light and other influences operating at the speed of light could no longer 'smooth out' the temperature of the sphere. This left just enough 'lumpiness' in the density to enable matter to congeal (under mutual gravitational attraction) into the sort of structures that we observe today.

If inflation continued for much longer at this rate, then $\dot{r}$ (and r for that matter) would have diverged quickly to (near) infinity and there would have been no structures like galaxies, clusters etc. But apparently the universe was saved from that fate by the fact that it cooled down during this massive expansion.

Theory has it that the vacuum 'supercooled' and then smoothly, but rapidly 'froze' into the classical vacuum, somewhat like supercooled water freezing abruptly into ice, where a lot of latent energy is released. In the case of the universe, that energy went into the creation of the elementary particles of matter. There was then only radiation, electrons and quarks, the latter being the building blocks of protons and neutrons.

Broadly speaking, this (possibly) is the expansion history of the universe for the first 10^{-32} seconds of it's life. At that stage the present observable universe had a size measured in metres and was expanding at one tremendous rate.

It was filled with all the radiation energy and the mass energy of the building blocks of the matter that we experience today. In the absence' of additional vacuum energy, the expansion rate $\dot{r}$ started to slow down, due to the extreme gravitational field caused by the energy content.

The expansion rate was however precisely balanced to the energy content of the early universe. Let us take the full expansion law, equation 14.9 of the previous chapter, repeated her for convenience:

$$\dot{a}/a = \bar{H}_0 \sqrt{\frac{1-\Omega}{a^2} + \frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4} + \Omega_v} \; .$$

Now simplify it fully for the conditions just after inflation, i.e., $\Omega = 1$, $\Omega_v = 0$. Also, since $a \ll 1$,* we can take $\Omega_r/a^4 \gg \Omega_m/a^3$, leaving

*Just after inflation ended, the expansion factor was $a \approx 10^{-27}$.

$$\left(\frac{\dot{a}}{\bar{H}_0}\right)^2 \cong \frac{\Omega_r}{a^2}. \tag{15.4}$$

The left side of the equation is proportional to expansion energy 'density', because expansion rate squared is proportional to energy (think about $\frac{1}{2}mv^2$) and $\bar{H}_0$ is the present expansion rate. This says that the early expansion energy density was proportional to the radiation energy density at that time.

But, how could radiation energy act as the 'braking' force, working against expansion, in a universe consisting mainly out of radiation? The answer is simply that any form of energy, be it mass energy, pressure energy, radiation energy, or whatever, will cause gravitational attraction.

Figure 15.3 shows the radius of our observable universe against time, in a log-log plot from the 'beginning' up to now. This type of plot magnifies the early times tremendously and compresses the present epoch. Only a small section on the right hand side of the plot represents the epoch from 'last scattering' (at ≈ 300,000 years) up to now (i.e., the visible universe). But it shows the inflationary epoch, with it's exponential expansion dramatically.

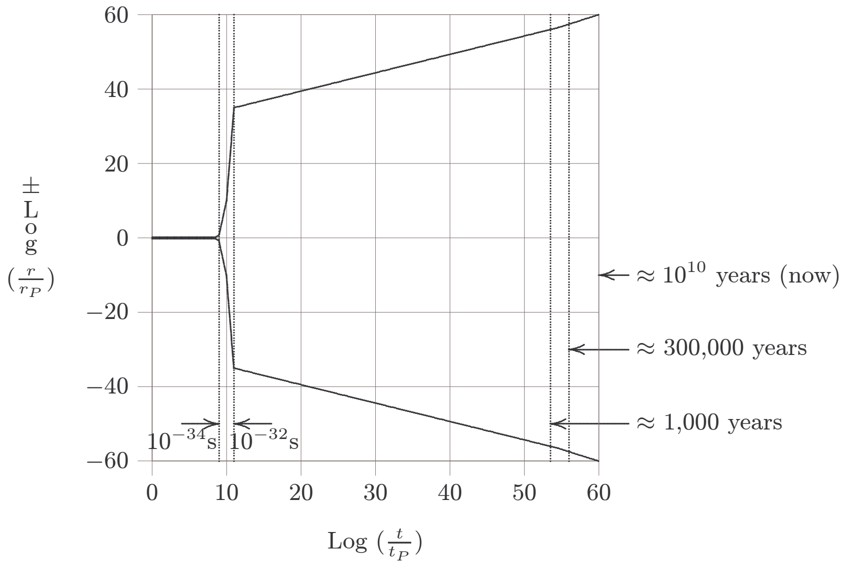

Figure 15.3: A log-log plot of radius of the observable universe against time from one Planck time unit until today. The rapid inflation lasted from about 10^{-34} to 10^{-32} seconds. After that the universe was radiation dominated up to about 1,000 years. Then the expansion curve changed to the matter dominated form that we presumably have today. See text for alternatives to that.

During the epoch from 10^{-32} seconds to around 1,000 years, radiation dominated in the density equation and the slope of the straight line is $\frac{1}{2}$, meaning $r_1/r_2 = (t_1/t_2)^{\frac{1}{2}}$. Thereafter, matter started to dominate and the slope of the line becomes $\frac{2}{3}$, meaning $r_1/r_2 = (t_1/t_2)^{\frac{2}{3}}$, the decreasing expansion rate of the standard model. This model ignores the possibility of vacuum energy becoming a factor in the density equation at a later stage.

It is important to note that in a log-log plot, a straight line does not represent linear expansion, but rather a power function, $r_1/r_2 = (t_1/t_2)^n$, where

n is constant. If lines are not straight, like during the inflationary epoch, then n grows with time and the situation is unstable.

When there is an abrupt change in the log-log slope, like at time 10^{-32} seconds, it does not mean that the expansion rate changes abruptly. It simply means accelerating expansion becomes decelerating expansion and the expansion rate $\dot{r}$ changes rapidly, yet smoothly at all times.

15.3 Is the expansion rate actually decreasing?

If the contribution of vacuum energy dropped to exactly zero after inflation ended (and remain zero), the log-log curve will today have a slope of $2/3$ and will remain so forever, which means that $\dot{r}$ will approach zero as the age of the universe tends to infinity.*

*A constant slope of less than unity on this log-log curve represents a parabola on a linear plot.

This is however not what today's astronomers observe. There are strong indications that the expansion rate in the matter dominated phase is not dropping at the rate that the standard model requires and it may even be increasing. The only plausible explanation is that some form of additional expansion energy is at work and this could possibly come from the fact that the vacuum contribution did not drop to exactly zero after inflation. So a mild form of inflation may still be present today.

As we have seen, vacuum energy is not diluted by the expansion of the universe and (if present) will eventually dominate the expansion, causing the log-log curve to flare out as shown in figure 15.4, meaning the expansion rate increases.

An expansion rate that first decreases and then increases makes the observed higher values of the Hubble constant H_0 compatible with an older universe—14 Gy or more. This is old enough to accommodate the apparent age of the oldest stars.

Apart from solving the 'age problem', the accelerating expansion rate apparently has little influence on us (and our descendants) here on Earth due to the timescales involved. It looks dramatic on the log-log scale because of the compression of time on the right hand side of figure 15.4.

However, 10^{14} years means that the universe is then ten thousand times older than it is today, or some hundred trillion years of age. What is important though is that apparently the slope of the log-log curve has recently* started to increase and is exceeding the unity value today, meaning the cosmic expansion rate is increasing.

*Perhaps as "recently" as 5 Gy ago.

The line drawn at 10^{14} years is thought to be roughly the boundary between

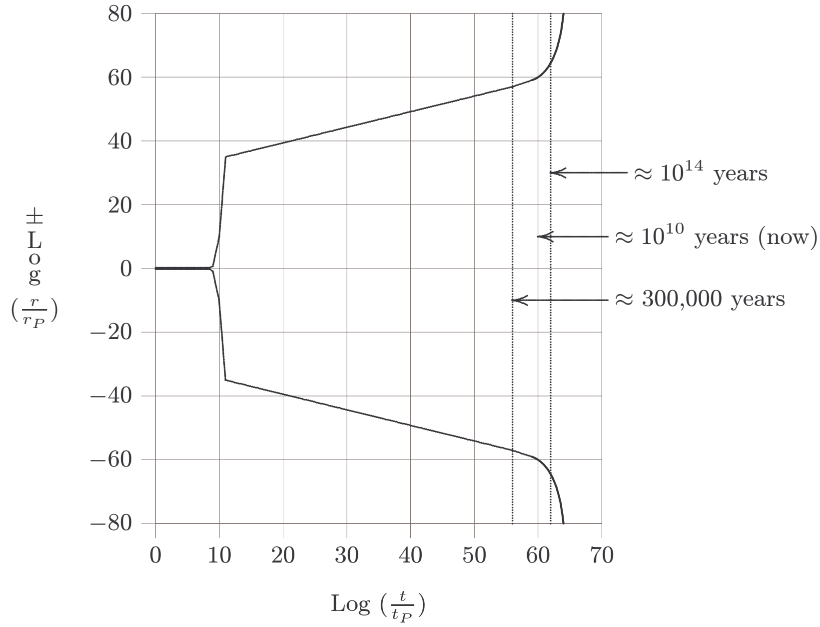

Figure 15.4: The radius against time if vacuum energy is still operating today. If present, the effect of vacuum energy will increase dramatically in the distant future, when the universe is thousands of times older than it is today. Theory has it that at 10^{14} years the universe will begin to degenerate into dead stars and black holes only.

the 'generative' and the 'degenerative' epochs. Most of the ordinary matter of the universe will become locked up in dead stars—white dwarfs, neutron stars or black holes.

Eventually, after a time greatly exceeding 10^{14} years, protons will decay, meaning white dwarfs and neutron stars will 'evaporate' into radiation and other elementary particles, so that only black holes will remain. An unimaginable long time after that, even black holes will become extinct due to Hawking radiation.*

*After Stephen Hawking, who proved this effect theoretically.

Because of the huge timescale, there will be no descendants of ours on Earth at 10^{14} years because our Sun is expected to 'flame out' in another 5 Gy or so, which is practically still on the "now" line of figure 15.4.

Should there be any observers left (somewhere in our galaxy) when the universe reaches the age of 10^{14} years, they will find it a much less exciting place for astronomy than it is today. All other superclusters would have drifted out of view. The increasing expansion rate would have surely taken them outside the observable universe for that time.

From the far-far future back to the present epoch. If we plot the expansion of the universe on a linear scale, the differences between a standard (no vacuum energy) expansion and a vacuum driven expansion is shown much more clearly for the present time. Figure 15.5 shows such a plot from the 'beginning' up to when the universe will be more than twice it's present age. Now the inflationary epoch and the other early epochs are lost due to the

Figure 15.5: The solid curve plots the expansion factor $a(t)$ against time for a flat universe ($\Omega = 1$), with vacuum energy today contributing 70% of the expansion energy. For Ho = 64 km/sec/Mpc, the present age of the universe will then be near 15 Gy (read off at $a = 1$). The plot is extended to more than twice the present age to show the increasing expansion rate more clearly. For comparison, the dotted curve is for a flat universe with no vacuum energy present today. See text for more details.

resolution, but the present is shown in proportion.

The solid curve is for when today's vacuum energy component makes up 70% of the total energy needed to make the universe flat. The dotted curve (part of a parabola) is for a flat universe with no vacuum energy component operating today—i.e., there are enough ordinary matter and dark matter to make the universe flat. The two curves have both been drawn for a present Hubble constant $H_0 = 64$ km/sec/Mpc or $\bar{H}_0 = 0.065$ Gy^{-1}.

Of significance is the fact that at $a = 1$, the slopes of the two curves are equal, meaning that the time varying Hubble constant $H(t) = \dot{a}/a$ is the same for the two curves. So where curves cut $a = 1$, it gives the present age of the universe for the two curves respectively.

For the assumed value of the Hubble constant it gives roughly 10 Gy for the matter only universe and close to 15 Gy for the universe with 70% vacuum energy. This shows graphically why vacuum aided expansion makes a specific value of the Hubble constant compatible with an older universe.

Something that is not so easy to understand is why the vacuum aided expansion curve lies below the the matter-only (dotted) curve for much of the time. The answer lies in the fact that the inflationary epoch 'delivered' a universe with the expansion rate precisely balanced with the amount of matter (visible and dark) that caused the slowing down effect through mutual gravitational attraction. For the vacuum aided expansion curve, the amount of matter must today be less than what it would have been for a matter-only universe—only 30% for the case plotted.

So during early years, when there was negligible vacuum effect, the amount of matter to be balanced by expansion rate must have been less in the same proportion. This 'lighter' universe would initially have expanded slower than the 'heavier' one. It can be easily deduced from the expansion law equations, given earlier.

More about the units and constants of the plots of the inflationary epoch

Working in units of seconds at these incredibly tiny time intervals is not a lot of fun. One solution is to convert time to units of Planck time (t_P), i.e. multiples of 10^{-43} seconds (the exact value is $t_P = 5.3906 \times 10^{-44}$ seconds). Time 10^{-43} seconds will then be 1 Planck time unit, time 10^{-42} seconds will be 10 Planck time units and so on. This leads to the simplest scheme of all - plot the radius r against $\log(t/t_P)$. Then 10^{-43} seconds will be expressed as zero time units, 10^{-42} seconds as one time unit and so on. Since r has magnitudes comparable to t, it is best to also express r in the same type of units, i.e. as $\log(r/r_P)$, where r_P is the radius expressed in Planck lengths, which is about 10^{-43} light-seconds. With these values it is convenient to work in log base 10 units and since we are interested in ratios, we can write the inflationary expansion equation as

$$\frac{r}{r_P} = \frac{b \times 10^{\alpha t}}{r_P}.$$

If we choose the constant $b = r_P$ (one Planck length), then b and r_P cancel out and we have the log-log curve for the inflationary expansion plot as

$$\log \frac{r}{r_P} = \alpha t.$$

The constant α depends on the inflationary model used. For the inflationary part of the plot, a value of $\alpha = 10^{34}$ per second was chosen so that time constant of inflation is 10^{-34} seconds (9 log-time units). This results in inflation happening between 9 and 11 log-time units, in the range given by most standard inflation models.

For the epochs after inflation ended (at 11 time units), the straight parts of the log-log curve is obtained by the slopes of the corresponding epoch: from 11 to about 53.5 time units (1,000 years), the slope is 1/2 and after that the slope is 2/3 (assuming that no vacuum energy is at work).

The approach followed here is really engineering-like! We accept what the scientists tell us without fully understanding the physics behind it and then apply it, albeit somewhat empirically! Although the analysis is not rigorous or precise, it gives us a reasonable idea of why the curves look like they do.