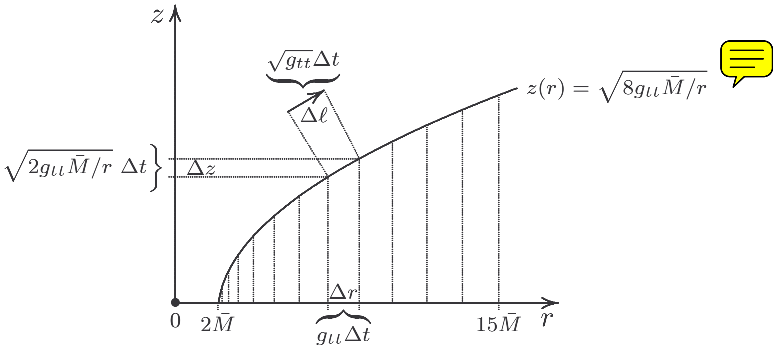

Figure 1.5: Proper radial distance increments ($\Delta\ell$) against coordinate radial distance increments (Δr), showing both gravitational time dilation (redshift) and space curvature. The segments $\Delta\ell$ represent the proper distance that light travel in time interval Δt, which become shorter closer to the origin due to gravitational time dilation. Then the gradient of the local space curve $z(r)$ causes the projection onto the coordinate radial axis to be 'compressed' further.

giving z as a function of r (after integration, with the expanded g_{tt})*

*An easy to follow derivation of $z(r)$ is given in [MTW], section 23.8.

$$z(r) = \sqrt{\frac{8g_{tt}\bar{M}}{r}} + \text{constant}, \tag{1.26}$$

as used in figure 1.5. This figure illustrates so much of the gravitational field around a static, spherically symmetric mass, that it warrants a closer look. Firstly, if space had no curvature, i.e., $z(r) =$ constant, there would still have been an apparent contraction in the radial direction, as observed by the distant observer, i.e.,

$$\Delta\ell = \sqrt{g_{tt}}\Delta t,$$

the distance that light propagates in coordinate time interval Δt.

If clocks slow down near the central mass, then so does the propagation of light, at least as viewed by the distant observer. A local observer cannot detect this 'slowing down' of light, because if your clock and your measuring rod* changes precisely in step, you cannot detect the 'slowing down'.

*All distance measurements are directly or indirectly based on the speed of light.

A distant observer can, in principle, detect the 'slowing down' of light by measuring the round trip travel time of light, beamed from a large distance to the mass and being reflected back from the surface of the mass.*

This is the effect of gravitational redshift alone. Because of the gradient of curved space, the effective movement of light in the radial direction is still 'slower' than that. From the gradient equation and since $dr^2 = d\ell^2 - dz^2$ (see figure 1.5), it is easy to show that

$$dr = \sqrt{g_{tt}}\ d\ell = g_{tt}\ dt, \tag{1.27}$$