

## Cosmic distances

The most useful cosmic distance measure is the comoving distance, which is the present proper distance to an object with observed redshift  $z_{now}$ . For the general case, the comoving distance is obtained by this integration:

$$D_{now} = \frac{978}{H_0} \int_0^{z_{now}} \frac{dz}{\sqrt{\Omega_k(1+z)^2 + \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_v}}$$

in units giga-light-years (Gly). Here  $H_0$  is the Hubble constant in units km/s/Mpc (978 is the conversion factor for  $H_0$  to  $\text{Gly}^{-1}$ ),  $\Omega_k$  the present curvature density parameter (which is very close to zero, meaning flat space),  $\Omega_m$  the mass density parameter (including dark matter),  $\Omega_r$  the radiation energy density parameter and  $\Omega_v$  the vacuum energy density (dark energy) parameter. Vacuum energy density is also denoted as  $\Omega_\Lambda$ , where  $\Lambda$  means Einstein's cosmological constant.

If the present expansion ratio  $z_{now}$  is applied after the integration, the original distance of the object at the time of emission is obtained:

$$D_{then} = \frac{D_{now}}{1 + z_{now}}$$

This is fairly obvious, because  $z + 1$  is inversely proportional to the expansion parameter, which tells us by what factor the universe has expanded in the time that the photons took to reach us.  $D_{then}$  is also approximately equal to the angular diameter distance of the source.

If the time variable expansion ratio  $(1 + z)$  is applied inside the integration, the distance that light had to travel to reach us from the source is obtained:

$$D_{light} = \frac{978}{H_0} \int_0^{z_{now}} \frac{dz/(1+z)}{\sqrt{\Omega_k(1+z)^2 + \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_v}}$$

This applies the varying expansion factor all along the path of a photon and adds up the fractional distances to obtain the real path length of the light.  $D_{light}$  is also called the lookback distance of an object with redshift  $z_{now}$ .

The curves were plotted with the following contemporary values:  $H_0 = 71$  km/s/Mpc,  $\Omega_k = 0$ ,  $\Omega_m = 0.27$ ,  $\Omega_r = 8.35 \times 10^{-5}$ ,  $\Omega_v = 0.73$ .